

MHD Stagnation Point Flow of Casson Nanofluid over a Stretching Sheet with effect Of Viscous Dissipation

T. Srinivasulu*¹, Shankar Bandari² and Chenna. Sumalatha³

**¹Department of Mathematics, M.V.S GDC, Mahabubnagar 509001,
Telangana, India.*

*^{2&3}Department of Mathematics, Osmania University, Hyderabad 500007,
Telangana, India.*

Abstract

This paper numerically analyzes MHD stagnation point flow of Casson nanofluid over a linear stretching sheet with the effect of viscous dissipation . The governing equations of the problem are transformed into non-linear ordinary differential equations by using similarity transformations. The resulting equations are solved numerically by using an implicit finite difference method known as Keller Box method. The effect of various physical parameters on the dimensionless velocity, dimensionless temperature and dimensionless concentration profile are showed graphically and discussed for the relative parameters. Present results are comparisons have been made with previously published work and results are found to be very good agreement. Numerical results for local skin friction, local Nusselt number and local Sherwood number are tabulated for various physical parameters.

Keywords: MHD, Stagnation-point, Linear stretching sheet, Viscous dissipation, Casson nanofluid.

NOMENCLATURE:

a	constant acceleration parameter
B_0	magnetic field
T	Temperature of the fluid in the boundary layer
C	concentration of the fluid in the boundary layer
T_w	stretching surface temperature
C_w	stretching surface concentration
T_∞	Ambient fluid temperature
C_∞	Ambient fluid concentration
u	velocity component along x-axis
v	velocity component along y-axis
u_w	velocity component at the wall
v_w	velocity component at the wall
ν	kinematic viscosity
σ	Density of fluid
α	Thermal diffusivity
D_B	Brownian diffusion coefficient
D_T	Thermophoresis diffusion coefficient
k	Thermal conductivity
Ec	Eckert number
M	Magnetic parameter
Pr	Prandtl number
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Le	Lewis number
β	Casson parameter
λ	Velocity ratio parameter
Cf_x	Local skin friction coefficient
Nu_x	Local Nusselt number
Sh_x	Local Sherwood number

1. INTRODUCTION:

Real fluids are two types namely Newtonian and Non-Newtonian fluid .A fluid obey the Newton law of viscosity is Newtonian fluid ,otherwise it is Non-Newtonian fluid. Many fluids in industries resemble non-Newtonian behavior. In non-Newtonian fluids, the relationship between stress and the rate of strain is not linear. Due to non-linearity between the stress and rate of strain for non-Newtonian fluids it is difficult to express all those properties of several non-Newtonian fluids in a single constitutive

equation. This has called on the attention of researchers to analyze the flow dynamics of non-Newtonian fluids. Consequently several non-Newtonian fluid models [22-27] have been proposed depending on various physical parameters. In 1959, Casson introduced Casson fluid model. If the shear stress is less than the applied yield stress on the fluid then Casson fluid act as a solid. If the shear stress is greater than the applied yield stresses then it act as a liquid. Fluids like honey, blood, soup, jelly, stuffs, slurries, artificial fibers are some Casson fluids. Krishnendu Bhattacharya[5] investigated MHD stagnation point flow of casson fluid and heat transfer over a stretching sheet in the presence of thermal radiation, in his observation the velocity boundary layer thickness for Casson fluid is larger than that of Newtonian fluid, the thermal boundary layer thickness decreases when Casson parameter decreases for $\beta < 1$ and increases when thickness increase for $\beta > 1$. Ibukum Sarah Oyelkin et.al[10] studied numerically the effects of thermal radiation, heat generation and combined effect of Soret and Dufour numbers on the Casson nanofluid over a unsteady stretching sheet by using Spectral Relaxation method. T.Vijayalaxmi et.al[14] analyzed the effects of inclined magnetic field, partial velocity slip and chemical reaction on casson nano fluid over a nonlinear stretching sheet and observed their study, increasing the values of Casson parameter leads to decreasing velocity profile but it is reverse in the case of temperature profile. Several other studies have addressed various aspects of Casson fluid[16-20].

Stagnation point is a point in the flow field where the local velocity of fluid particle is zero. The flow near stagnation point has attracted the attention of many investigators during the past several decades, in view of its wide range of applications such as cooling of nuclear reactors of electronic devices by fans and many hydrodynamic processes. Mahapatra and Gupta [1&28] investigated the heat transfer effect on stagnation point flow towards a stretching sheet in the presence of viscous dissipation effect. Later they studied the influence of heat transfer stagnation point flow past a stretching sheet. In their study, boundary layer is formed when the stretching velocity less than a free stream velocity and an inverted boundary layer is formed when the stretching velocity exceeds the free stream velocity. Wubshet Ibrahim .et.al [2] investigated numerically by using Runge-Kutta fourth order method, heat transfer characteristics of nanofluid in the presence of magnetic field at near to stagnation point flow over a stretching sheet. Hayat .et.al [3] analyzed MHD flow of micro polar fluid near a stagnation point towards a nonlinear stretching sheet. Imran Anwar .et.al [6] numerically studied MHD stagnation-point flow of a nanofluid over an exponential stretching sheet with the effect of radiation by using Keller Box method. Mohd Hafizi Mat Yasin [9] used the Runge-Kutta Fehlberg method of solution to study the steady two dimension stagnation -point flow over a permeable stretching sheet and heat transfer in the presence of magnetic field with the effects of viscous dissipation, joul heating and partial velocity slip. Several other studies have addressed various aspects of MHD stagnation-point flow of fluids[5,7,8,11,12,13&14].

The study of boundary layer flow over a stretching sheet has many applications in industrial processes such as paper production, wire drawing, glass fiber production etc. Steady laminar flow and heat transfer of a nanofluid over a flat plate surface is

numerically investigated by Rana and Bhargava [4]. Winifred Nduku Mutuku [7] studied MHD boundary layer flow of nanofluid with effect of viscous dissipation and observed that local Sherwood number increases with an increase in Eckert number. Dufour and Soret effects on heat and mass transfer of a Casson nanofluid was investigated by Ibukun Sarah Oyelakin [10].

Motivated by above investigations on Casson nanofluid and its wide applications, the objective of the present study is to analyze MHD stagnation-point flow over a stretching sheet with the effect of viscous dissipation. In addition to this, the effects of governing parameters such as magnetic parameter, velocity ratio parameter, Eckert number, Prandtl number, Lewis number, Brownian motion parameter, Thermophoresis parameter and Casson parameters also analysed.

2. MATHEMATICAL FORMULATION:

Consider a two dimensional steady, viscous and incompressible MHD stagnation point flow of Casson nanofluid over a linear stretching sheet with the plane $y=0$ and the being confined to $y>0$ and y coordinate is normal to the plane/sheet under the effect of viscous dissipation kept at a constant temperature T_w and concentration C_w . The ambient temperature and concentration are T_∞ and C_∞ respectively. The velocity of the stretching sheet is $u_w(x)=ax$ (where $a > 0$ is the constant acceleration parameter) and the velocity of the ambient fluid is $U_\infty=bx$ (where $b>0$). The fluid is electrically conducting under the influence of magnetic field $B(x)=B_0$ normal to the stretching sheet. The induced magnetic field is assumed to be small compared to the applied magnetic field and is neglected. The physical flow and co-ordinate system is shown in the Fig.1. The rheological equation of state for an isotropic and incompressible flow of Casson fluid [Nakamura and Sawada [14], Mustapaet.al[17] is given by.

$$\tau_{ij} = \begin{cases} 2 \left(\mu_B + \frac{p_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\ 2 \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \pi < \pi_c \end{cases}$$

Where μ_B and p_y are the plastic dynamic viscosity, yield stress of the fluid respectively. Similarly π is the product of the component of deformation rate with itself, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i,j)-th component of the deformation rate and π_c is a critical value of this product based on non-Newtonian model.

Under the above boundary conditions, the governing equations of boundary layer equations are

The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

The momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{\partial U_\infty}{\partial x} + \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B_0^2}{\rho_f} (U_\infty - u) \tag{2}$$

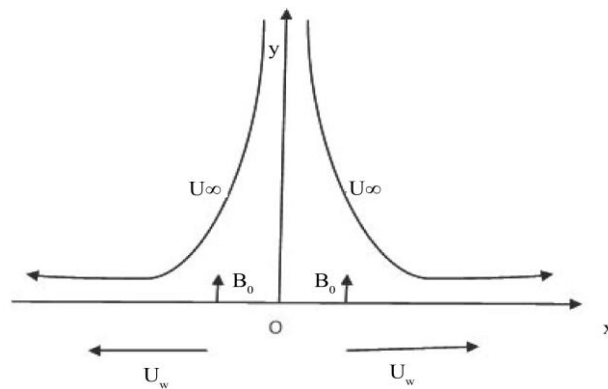
The energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial c}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y}\right)^2 \right] + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The nanoparticle concentration equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Where u, v are velocity components along x -axis and y -axis respectively. $U_\infty, \alpha, \nu, \rho, C_p, (\rho c)_f, D_B, D_T, \beta$ and τ are, freestream velocity, Thermal diffusivity, kinematic viscosity, mass density, specific heat, effective heat capacity of the nanoparticle material, heat capacity of the fluid, Brownian diffusion coefficient, thermophoresis diffusion coefficient, casson parameter and a parameter defined as the ratio of effective heat capacity of the nanoparticle material to heat capacity of the fluid respectively.



The associated boundary conditions are

$$\begin{aligned} \text{At } y=0: \quad & u \rightarrow U_w = ax; & v=0; & T=T_w; & C=C_w \\ \text{At } y \rightarrow \infty: & u \rightarrow U_\infty = bx; & T \rightarrow T_\infty; & C \rightarrow C_\infty \end{aligned} \tag{5}$$

Introduce the following similarity transformations

$$\left. \begin{aligned} \eta = y \sqrt{\frac{a}{\nu}} \quad ; \quad \psi = \sqrt{a\nu} \, x f(\eta) \quad ; \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad ; \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \right\} \tag{6}$$

Where ψ denotes stream function and is defined as $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ and $f(\eta)$ is a dimensionless stream function, ϕ is dimensionless concentration function and θ is dimensionless temperature function and η is similarity variable. After using similarity transformations, the governing equations (2)-(4) are reduced to the ordinary differential equations as follows:

$$\left(1 + \frac{1}{\beta}\right) f''' + ff'' - (f')^2 - Mf' + (M\lambda + \lambda^2) = 0 \quad (7)$$

$$\frac{1}{Pr} \theta'' + Nb \theta' \phi' + \left(1 + \frac{1}{\beta}\right) Ec (f'')^2 + \theta' f + Nt (\theta')^2 = 0 \quad (8)$$

$$\phi'' + Le (f\phi') + \frac{Nt}{Nb} \theta'' = 0 \quad (9)$$

The associative boundary conditions becomes

$$\left. \begin{array}{l} \text{At } y=0: \quad f'(0)=1; \quad f(0)=1; \quad \theta(0)=1; \quad \phi(0)=1 \\ \text{At } y \rightarrow \infty: \quad f'(\infty)=\lambda; \quad \theta(\infty)=0; \quad \phi(\infty)=0 \end{array} \right\} \quad (10)$$

Where the governing parameters defined as:

$$\left. \begin{array}{l} M = \frac{\sigma B_o^2}{\rho_f a}; \quad \lambda = \frac{b}{a}; \quad Pr = \frac{\nu}{\alpha}; \quad Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu} \\ Nt = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu}; \quad Ec = \frac{U_w^2}{C_p (T_w - T_\infty)}; \quad Le = \frac{\nu}{D_B} \end{array} \right\} \quad (11)$$

Here $M, Pr, \lambda, Nb, Nt, Le$ and Ec denote Magnetic parameter, Prandtl number, Velocity ratioparameter, the Brownian motion parameter, the Thermophoresis parameter, the Lewis number and Eckert number respectively.

The quantities of practical interest in this study Local skin friction co-efficient C_{fx} , the Local Nusselt number Nu_x and Local Sherwood number Sh_x which are defined as follows :

$$C_{fx} = \frac{\mu_f}{\rho U_w^2} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \quad (12)$$

Where k is the thermal conductivity of the nanofluid and q_w, q_m are heat and mass fluxes at the surface respectively and define as follows

$$q_w = - \left[\frac{\partial T}{\partial y} \right]_{y=0} \qquad q_m = - D_B \left[\frac{\partial C}{\partial y} \right]_{y=0} \qquad (13)$$

Substituting equations (6) into (12) and (13) we obtain

$$\text{Re}_x^{1/2} C_{fx} = f''(0), \quad \text{Re}_x^{-1/2} Nu_x = -\theta'(0), \quad \text{Re}_x^{-1/2} Sh_x = -\phi'(0) \qquad (14)$$

Where $\text{Re}_x = \frac{u_w x}{\nu}$ is the local Reynolds number

3. NUMERICAL METHOD:

The non linear ordinary differential equations (7)-(9) together with boundary conditions (10) are solved numerically by an implicit finite difference scheme namely the Keller box method as mentioned by Cebeci and Bradshaw ²¹. According to Vajravelu et al ²², to obtain the numerical solutions, the following steps are involved in this method.

- Reduce the ordinary differential equations to a system of first order equations.
- Write the difference equations for ordinary differential equations using central differences.
- Linearize the algebraic equations by Newtons method, and write them in matrix vector form.
- Solve the linear system by the block tri-diagonal elimination technique.

The accuracy of the method is depends on the appropriate initial guesses. We made an initial guesses are as follows.

$$f_0(\eta) = (1 - \lambda) + \lambda x + (\lambda - 1)e^{-x} \quad \theta_0(\eta) = e^{-x}, \quad \phi_0(\eta) = e^{-x}.$$

The choices of the above initial guesses depend on the convergence criteria and the transformed boundary conditions of equation (9) and (10). The step size 0.1 is used to obtain the numerical solution with four decimal place accuracy as the criterion of convergent.

4. RESULT AND DISCUSSIONS:

The nonlinear differential Equations [7], [8] and [9] with boundary conditions [10] are solved numerically by using Implicit finite difference method known as Keller box method Cebeci and Bradsha²¹ and vajravelu.et.al²². Table I and II shows the comparison of the data produced by the present method and that T.Ray Mahapatra , AS.Gupta¹ and T.Hayat,T.Javed and Z.Abbas³ . The result show excellent agreement among data.

Table I. The comparison of values of Skin friction coefficient $-f''(0)$ when $Pr=1, M = Ec = Nb = Nt = Le = 0$ and $\beta=99999$

λ	Present result	Mahapatra[1]	Hayat[3]
0.01	0.9987	---	0.9983
0.1	0.9697	0.9694	0.96954
0.2	0.9184	0.9181	0.91813
0.5	0.6676	0.6673	0.66735
2	2.0201	2.0175	2.01767
3	4.7393	4.7293	4.72964

Table II. Comparison of Nusselt number when $Pr = 1, M = Ec = Nb = Nt = Le = 0$ and $\beta=999999$

Pr	λ	Present	Hayath[3]
1	0.1	0.6020	0.6021
	0.2	0.6244	0.6244
	0.3	0.6473	0.6924
1.5	0.1	0.7768	0.7768
	0.2	0.7972	0.7971
	0.3	0.8193	0.8193

Table III. Computed the values of skin friction coefficient, Local Nusselt number and Sherwood number for various values parameters.

Pr	M	λ	Ec	Nb	Nt	Le	B	Skin fric	Nusselt number	Sherwood Number
1	1	0.1	0.1	0.1	0.1	1	1	0.9342	0.4940	0.3266
								0.9342	0.6993	0.174
								0.9342	0.8367	0.0666
	1	0.1	0.1	0.1	0.1	1	1	0.9342	0.4940	0.3266
								1.1301	0.448	0.3199
								1.2968	0.411	0.3186
1	1	0.1	0.1	0.1	0.1	1	1	0.9342	0.4940	0.3266
		0.5						0.5885	0.6222	0.3880
		0.9						0.1301	0.7160	0.4389
		1.3						0.424	0.7698	0.5071
1	1	0.1	0.1	0.1	0.1	1	1	0.9342	0.4940	0.3266
			0.5					0.9342	0.2186	0.5738
			0.9					0.9342	0.0592	0.8215
1	1	0.1	0.1	0.1	0.1	1	1	0.9342	0.4940	0.3266
				0.2				0.9342	0.4665	0.4803
				0.3				0.9342	0.44	0.5312

1	1	0.1	0.1	0.1	0.1	1	1	0.9342	0.4940	0.3266
					0.2			0.9342	0.4773	0.0693
					0.3			0.9342	0.4613	0.1628
1	1	0.1	0.1	0.1	0.1	1	1	0.9342	0.4940	0.3266
						2		0.9342	0.484	0.7139
						3		0.9342	0.4793	1.0012
1	1	0.1	0.1	0.1	0.1	1	1	0.9342	0.4940	0.3266
							2	1.0787	0.4794	0.3020
							3	1.1482	0.4729	0.2922

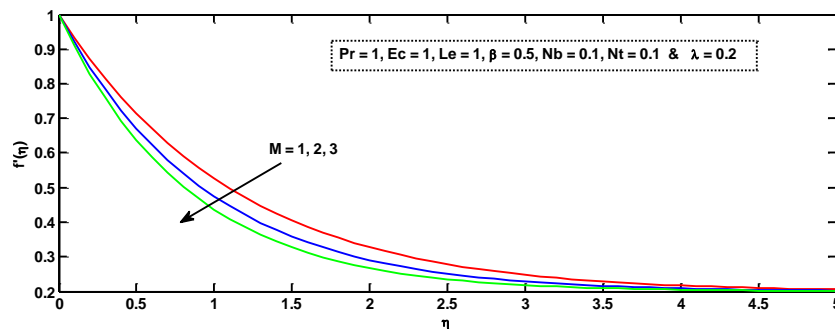


Fig.2. Effect of magnetic parameter M on velocity profile.

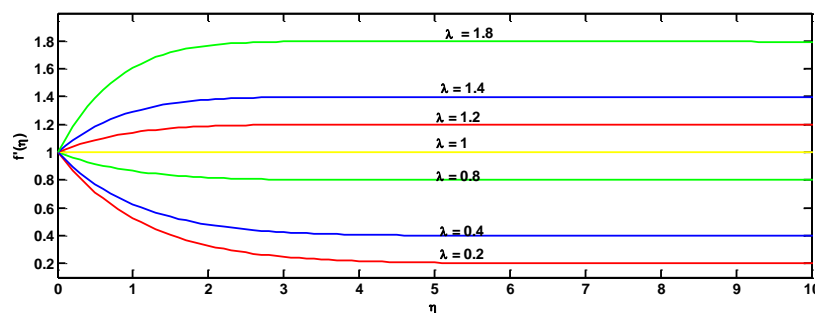


Fig.3 Effect of Velocity ratio parameter λ on velocity profile.

when $Pr = Ec = M = Le = 1$; $Nb = Nt = 0.1$ and $\beta = 0.5$

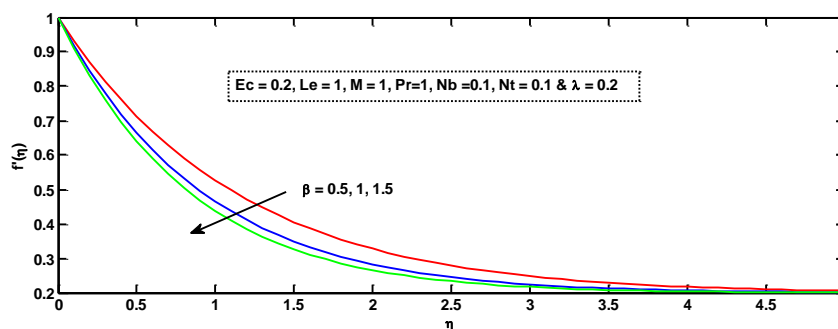


Fig.4 Effect of casson parameter β on velocity profile.

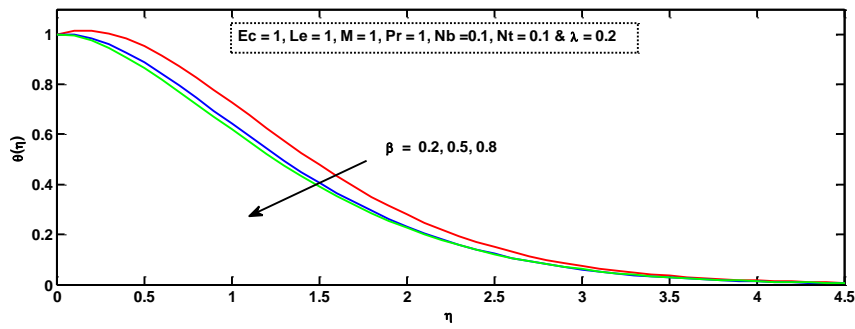


Fig.5 Effect of casson parameter β on temperature profile.

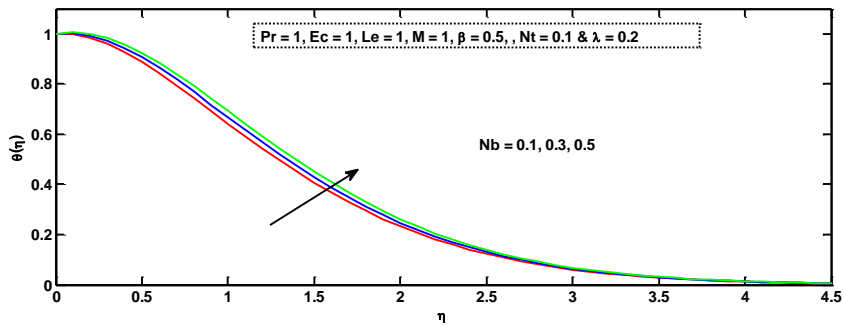


Fig.6 Effect of Nb on temperature profile.

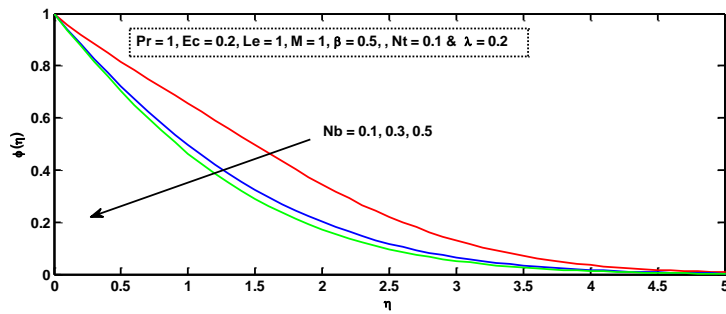


Fig.7 Effect of Nb on concentration profile.

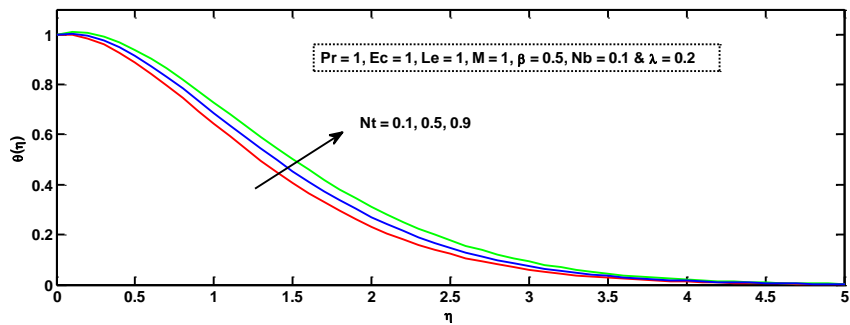


Fig.8 Effect of Nt on temperature profile.

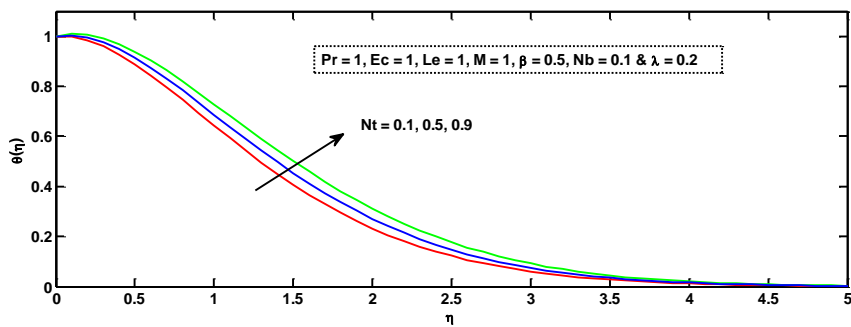


Fig.9 Effect of Nt on concentration profile.

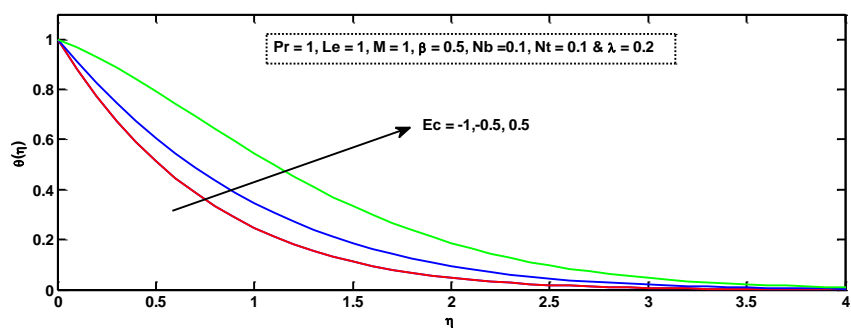


Fig.10 Effect of Eckert number Ec on Temperature profile.

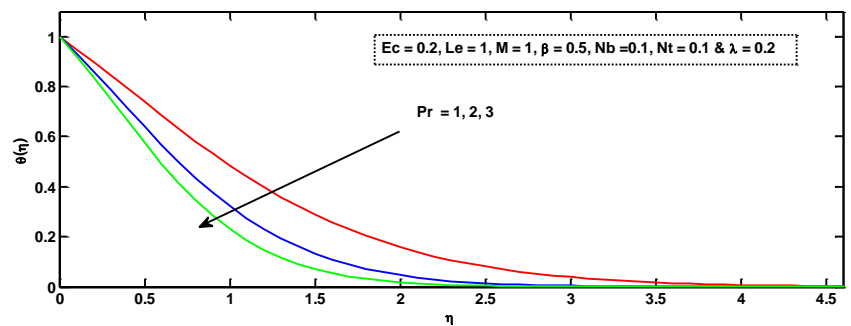


Fig.11 Effect of Prandtl number Pr on Temperature profile.

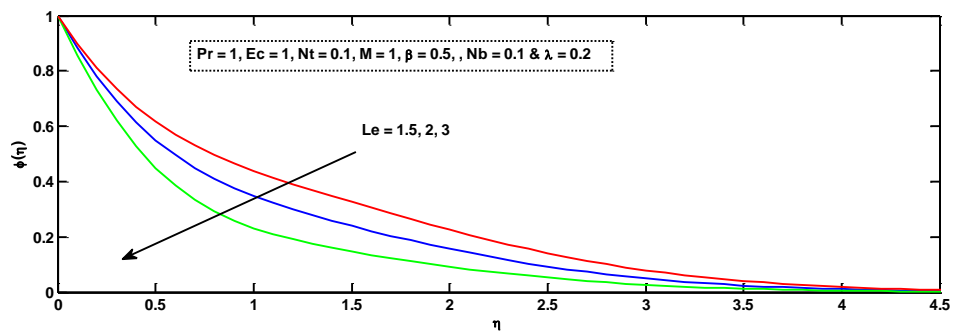


Fig.12 Effect of Lewis number Le on concentration profile.

Fig 2.shows the effect of magnetic parameter M on the velocity graph for various values of M . The presence of transverse magnetic field sets in Lorentz force which results in retarding force on the velocity field. Therefore as the values of M increase, so does the retarding force and hence the velocity decrease when $\lambda = 0.2$. The flow has boundary layer structure and the boundary layer thickness decreases as the values of M increase. Fig 3.shows the effect of velocity ratio parameter on the velocity graph. When the velocity of stretching sheet exceeds the free stream velocity (i.e. $\lambda = b/a < 1$), the velocity of the fluid and boundary thickness increase with an increase in λ . When the free stream velocity exceeds the velocity of stretching sheet (i.e. $\lambda = b/a > 1$), in this case the flow velocity increases and the boundary layer thickness decreases with an increase in λ . When the velocity of stretching sheet is equal to the free stream velocity, there is no boundary layer thickness of Casson nanofluid near the sheet.

Fig 4 and Fig 5.shows the effect of casson parameter (β) on velocity and temperature graphs for different values of β . It is observed that for increasing values of β the velocity profile decreases. Due to increase of β , the yield stress τ_y reduces and hence the momentum boundary layer thickness decreases.

Fig.6 the usual decay occurs to the temperature profiles for all values of Nb considered, and the thermal boundary layer thickness increases rapidly for large values of Nb . It is observed that the effect of Nb on the nanoparticle concentration profile $\phi(\eta)$ is in the opposite manner to that of temperature profiles $\theta(\eta)$ as illustrated in Fig. 7. It is apparent from Figs. 6 and 7 that nanoparticle concentration is decreasing as Nb increasing. It seems that the Brownian motion acts to warm the fluid in the boundary layer and at the same time exacerbates particle deposition away from the fluid regime to the surface which resulting in a decrease of the nanoparticle concentration boundary layer thickness for both solutions.

Figs. 8 and 9 present typical profiles for temperature and concentration for various values of thermophoresis parameter (Nt). It is observed that an increase in the thermophoresis parameter (Nt) leads to increase in both fluid temperature and nanoparticle concentration. Thermophoresis serves to warm the boundary layer for low values of Prandtl number (Pr) and Lewis number (Le). So, we can interpret that the rate of heat transfer and mass transfer decrease with increase in Nt . Fig 10 shows that the effect of Eckert number on temperature profile. temperature increase with an increase in Eckert number. The viscous dissipation produces heat due to drag between the fluid particles and this extra heat causes an increase of the initial fluid temperature.

The effect of Prandtl number Pr on the heat transfer process is shown by the Fig.11. This figure reveals that an increase in Prandtl number (Pr) results in a decrease in the temperature distribution, because, thermal boundary layer thickness decreases with an increase in Prandtl number (Pr). In short, an increase in the Prandtl number means slow rate of thermal diffusion. The graph also shows that as the values of Prandtl

number Pr increase, the wall temperature decreases. The effect of Prandtl on a nanofluid is similar to what has already been observed in common fluids qualitatively but they are different quantitatively. Therefore, these properties are inherited by nanofluids. Fig12 show the effect of Lewis number(Le) on concentration graph. The thickness of the boundary layer decrease with an increase in Le .

Table III: shows the variation of Skin friction coefficient $-f''(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$ for various values of parameters $M, Pr, Le, Nb, Nt, \lambda, Ec$ and β . Nusselt number & Sherwood number are generally used as the heat transfer rate and mass transfer rate at the surface of stretching sheet respectively. Skin friction coefficient values increases with the values of magnetic parameter, lewis number and casson parameter. Nusselt number increases with an increase in prandtl number, velocity ratio parameter and decreases in M, Nb, Nt, Le, β and Ec . Sherwood number values are increase with an increase in Nb, Nt, λ and Le .

4. CONCLUSIONS:

In the present numerical study, MHD stagnation point flow over a linear stretching sheet with the effect of viscous dissipation. The governing partial differential equations are transformed into ordinary differential equations by using a similarity transformations, which are then solved numerically using implicit finite difference method. The effect of various governing parameters namely magnetic parameter, velocity ratio parameter, Eckert number, casson parameter, Brownian motion parameter, thermophoresis parameter, Prandtl number and Lewis number on the velocity, temperature and concentration profile are shown graphically, presented and discussed. Numerical results for the skin friction, local Nusselt number and local Sherwood number are presented in tabular form. The main observation of the present study is as follows.

- Nusselt number increase when Pr and λ increase while Nusselt number decrease when Nb, Nt, Ec, Le, β and M increase.
- Sherwood number increases when Nt, Nb, Le, Ec and λ increase, while decrease when Pr and M increase.
- Skin friction coefficient increase when M, λ and β increase.
- Temperature profile increases with increase the values of Nt, Nb and Ec .
- concentration profile decreases when the values of Le and Nb increase.
- velocity profile decreases when M and β increase.

ACKNOWLEDGMENTS:

The author T. Srinivasulu wishes to express their thanks to University Grants Commission (UGC), India, for awarding Faculty Development Programme (FDP).

REFERENCES

- [1] T.Ray mahapatra and A.S.Gupta ,Heat transfer flow towards a stretching sheet ,Heat and Mass transfer 38((2002)517-521.
- [2] Wubset Ibrahim,Bandari Shankar ,Mahantesh,M.Nandeppanavar,MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet,International journal of Heat and Mass transferr56(2013)1-9.
- [3] T.Hayat,T.Javed ,Z.Abbas ,MHD flow of a micropolar fluid near a stagnation –point towards a non-linear stretching surface ,Non-linear Analysis :Real World Application 10(2009)1514-1526.
- [4] P.Rana ,R.Bhargava Flow and heat transfer of a nanofluid over a non-linear stretching sheet:A numerical study.common Nonlinear Sci Numer Simulat 17(2012)212-226.
- [5] Krishnendu Bhattacharya,MHD stagnation –point flow of casson fluid and heat transfer over a stretching sheet with thermal radiation,Hindawi Publishing Corporation ,Journal of Thermodynamics volume 2013 Article ID 169674.
- [6] Imran Anwar,Sharidan Shafie and Mohd Zuki Salleh,Radiation effect on MHD stagnation –point flow of a nanofluid over an exponentially stretching sheet,Walailak J Sci & Tech 2014;11(7):569-591.
- [7] Winifred Nduku Mutuku ,MHD non-linear flow and heat transfer of nanofluids past a permeable moving flat surface with thermal radiation and viscous dissipation, Universal journal of Fluid Mechanics 2 (2014).55-68.
- [8] M.Subhas Abel.Monayya Mareppa ,Jagadish V Tawade ,Stagnation point flow of MHD nanofluid over a stretching sheet with effect of heat source/ sink ,momentum , thermal and solutal slip,IJSR ,volume 3Issue 6 june 2014 ISSN NO 2277-8179.
- [9] Mohd Hafizi Msat Yasin ,Annuar Ishak and Ioan Pop,MHD stagnation point flow and heat transfer with effects of viscous dissipation ,joul heating and partial velocity slip,SCIENTIFIC REPORTS /5:17848/DOI:10,103/B/strep17848.
- [10] Ibukun Sarah Oyelakin ,Sabyasachi Mondal,Precious Sibanda ,Unsteady casson nano fluid over a stretching sheet with thermal radiation ,convective and slip boundary conditions ,Alexandria Engineering Journal (2016)55,1025-1035.
- [11] B.K.Mahatha ,R.Nandkeolyar ,G.Nagaraju,M.Das,MHD stagnation point flow of a nanofluid with velocity slip,Non-linear radiation and Newtonian heating,Procedia Engineering 127(2015)1010-1017.
- [12] Shankar Bandari,and and Yohannes Y,MHD stagnation point flow and heat transfer of nanofluids towards a permeable stretching sheet with effects of thermal radiation and viscous dissipation ,SASEC2015,third southern African Conference 11-13 May 2015 Kruger National park ,South Africa.
- [13] G.Vasumathi J.Anand Rao and B.Shankar ,MHD stagnation point flow and heat transfer of a nanofluid over a non-isothermal stretching sheet in porous

- medium, Physical science international journal 12(4):1-11,2016 Article no.PSIJ .29926 ISSN:2348-0130
- [14] Nakamura, M. and Sawada, T. Numerical study on the flow of a Non-Newtonian fluid through an Axisymmetric stenosis. *Journal of Biomechanical Engineering*, 110, 137-143 (1988).
- [15] Kai-Long Hsiao, Stagnation electrical MHD nanofluid mixed convection with slip boundary conditions on a stretching sheet, *Applied Thermal Engineering* 98(2016)850-861.
- [16] Ch. Vittal, M. Chenna Krishna Reddy, M. Monica, Stagnation point flow of a MHD Powell–Eyring fluid over a nonlinearly stretching sheet in the presence of heat source/sink. *Journal of Progressive Research in Mathematics (JPRM)* ISSN:2395-0218(2016).
- [17] Mustafa, M., Hayat, T., Pop, I. and Aziz, A. (2011) Unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate. *Heat Transfer*, 40, 563-576.
- [18] Monica Medikare, Sucharitha Joga, Kishore Kumar Chidem, MHD stagnation point flow of a Casson fluid over a nonlinear stretching sheet with viscous dissipation, *American Journal of Computational Mathematics*, 2016, 6, 37-48.
- [19] M. Tamoor M. Waqas, M. Ijaz Khan Ahmed Alsaedi, T. Hayat, MHD flow of a Casson fluid over a stretching cylinder, *Results in Physics* (2017).
- [20] M. Mustafa and Junaid Ahmad Khan, Model flow of Casson nanofluid past a non-linear stretching sheet considering magnetic field effects. *AIP Advances* 5, 077148 (2015).
- [21] T. Cebeci and P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer*, Springer-Verlag, New York, 1984.
- [22] Vajravelu, K., Prasad, K.V. and Ng, C.-O. (2013) Unsteady Convective Boundary Layer Flow of a Viscous Fluid at a Vertical Surface with Variable Fluid Properties. *Nonlinear Analysis: Real World Applications*, 14, 455-464.
- [23] Fox, V.G., Erickson, L.E. and Fan, L.T. (1969) The Laminar Boundary Layer on a Moving Continuous Flat Sheet Immersed in a Non-Newtonian Fluid. *AIChE Journal*, 15, 327-333. <http://dx.doi.org/10.1002/aic.690150307>
- [24] Wilkinson, W. (1970) The Drainage of a Maxwell Liquid Down a Vertical Plate. *Chemical Engineering Journal*, 1, 255-257. [http://dx.doi.org/10.1016/0300-9467\(70\)80008-9](http://dx.doi.org/10.1016/0300-9467(70)80008-9)
- [25] Djukic, D.S. (1974) Hiemenz Magnetic Flow of Power-Law Fluids. *Journal of Applied Mechanics*, 41, 822-823. <http://dx.doi.org/10.1115/1.3423405>
- [26] Rajagopal, K.R. (1980) Viscometric Flows of Third Grade Fluids. *Mechanics Research Communications*, 7, 21-25. [http://dx.doi.org/10.1016/0093-6413\(80\)90020-8](http://dx.doi.org/10.1016/0093-6413(80)90020-8)
- [27] Rajagopal, K.R. and Gupta A.S. (1981) On a Class of Exact Solutions to the Equations of Motion of a Second Grade Fluid. *International Journal of Engineering Science*, 19, 1009-1014. <http://dx.doi.org/10.1016/0020->

7225(81)90135-X

- [28] Mahapatra .T and Gupta,A.S,(2001) Magnetohydrodynamic stagnation –point flow towards a stretching sheet ,Act Mechanica ,152,191-196.